Physics Letters B 728 (2014) 634–638

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The high energy neutrino nuisance at a medium baseline reactor experiment

Emilio Ciuffoli [a,b], Jarah Evslin [a,b], Xinmin Zhang [a,b]

[a] Theoretical physics division, Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(4), Beijing 100049, PR China
[b] Theoretical Physics Center for Science Facilities (TPCSF), Chinese Academy of Sciences, PR China

ARTICLE INFO

Article history:
Received 9 May 2013
Received in revised form 14 December 2013
Accepted 17 December 2013
Available online 27 December 2013
Editor: G.F. Giudice

ABSTRACT

10 years from now medium baseline reactor experiments will attempt to determine the neutrino mass hierarchy from quantities associated to the Fourier transformed neutrino spectra. Recently Qian et al. have claimed that this goal may be impeded by the strong dependence of these quantities on the reactor neutrino flux and on slight variations of $|\Delta M^2_{32}|$. We demonstrate that this effect results from a spurious dependence of the quantities on the very high energy (8+ MeV) tail of the reactor neutrino spectrum. This dependence is spurious because the high energy tail depends upon decays of exotic isotopes and is insensitive to the mass hierarchy. An energy-dependent weight in the Fourier transform eliminates this spurious dependence without decreasing the chance of correctly determining the hierarchy.

Last year the Daya Bay [1,2] and RENO [3] experiments demonstrated beyond any reasonable doubt that θ_{13} is as much as an order of magnitude larger than had been suspected several years ago, a discovery recently confirmed by T2K [4]. This large value of θ_{13} implies that 1–3 reactor neutrino oscillations may be observed at medium baselines, which we define to be 40–80 km. The medium baseline neutrino spectrum may then be used to determine the neutrino mass hierarchy [5]. Such experiments are now not only practical but indeed they will be performed within the next decade [6–8].

How does this determination work? With each fission chain, a nuclear reactor emits on average $6\bar{\nu}_e$'s in essentially random and isotropically distributed directions. The $\bar{\nu}_e$'s are detected via inverse beta decay upon their interaction with free (not bound to other nucleons) protons in a detector. Some of the $\bar{\nu}_e$'s oscillate into other flavors, providing an energy-dependent reduction of the flux which depends on the leptonic mixing angles θ_{12} and θ_{13}, on the neutrino mass differences and in particular on the neutrino mass hierarchy. In all, the $\bar{\nu}_e$ survival probability is

$$P_{ee} = \sin^4(\theta_{13}) + \cos^4(\theta_{12})\cos^4(\theta_{13}) + \sin^4(\theta_{12})\cos^4(\theta_{13}) + \frac{1}{2}(P_{12}+P_{13}+P_{23}),$$

$$P_{12} = \sin^2(2\theta_{12})\cos^4(\theta_{13})\cos\left(\frac{\Delta M^2_{21}L}{2E}\right),$$

$$P_{13} = \cos^2(\theta_{12})\sin^2(2\theta_{13})\cos\left(\frac{|\Delta M^2_{31}|L}{2E}\right),$$

$$P_{23} = \sin^2(\theta_{12})\sin^2(2\theta_{13})\cos\left(\frac{|\Delta M^2_{32}|L}{2E}\right). \tag{1}$$

The largest contribution to the depletion is caused by the P_{12} term in Eq. (1). At a medium baseline this corresponds to a single, broad dip in the measured neutrino spectra. On the other hand P_{13} and P_{23}, which we refer to collectively as 1–3 oscillations, provide a fine structure of small oscillations in the observed spectrum. Of these, the amplitude of P_{13} is greater than that of P_{23} by a factor of $\cot^2(\theta_{12}) \sim 2$, so P_{23} provides a perturbation to the P_{13} oscillations, which on their own would have been periodic in $1/E$. As the frequencies of P_{23} and P_{13} are slightly different, the fine structure, which consists of the sum of these two oscillations, is not quite periodic in $1/E$ [9]. On the contrary as the fine structure is the sum of two similar frequencies it exhibits a beating pattern, with a single beat visible in the spectrum at a medium baseline. It is the direction of the beating[1] which determines which frequency is greater. As one frequency is proportional to $|\Delta M^2_{31}|$ and the other to $|\Delta M^2_{32}|$, this direction determines whether $|\Delta M^2_{31}|$ is greater

[1] The direction of the beating corresponds to be whether, in each period, the intermediate peaks of $P_{13}+P_{23}$ are systematically at higher or lower energies than the peaks of P_{13}.

E-mail addresses: ciuffoli@ihep.ac.cn (E. Ciuffoli), jarah@ihep.ac.cn (J. Evslin), xmzhang@ihep.ac.cn (X. Zhang).

0370-2693 © 2013 The Authors. Published by Elsevier B.V. Open access under CC BY license.
http://dx.doi.org/10.1016/j.physletb.2013.12.045

than $|\Delta M^2_{32}|$, corresponding to the normal hierarchy, or $|\Delta M^2_{32}|$ is greater than $|\Delta M^2_{31}|$, corresponding to the inverted hierarchy.

This deviation from periodicity in the fine structure of the observed spectrum determines the hierarchy. More quantitatively, let E_n be the energy of the peak in the oscillated neutrino spectrum corresponding to neutrinos which have oscillated n times between the reactor and the detector. Note that higher values of n correspond to lower values of energy. It was shown in Ref. [10] that the inverse energies of the first 10 peaks are indeed periodic to within experimental error and indeed are well approximated by 2 flavor neutrino oscillation with an effective mass difference of [11]

$$\Delta M^2_{\text{eff}} = \cos^2(\theta_{12})\left|\Delta M^2_{31}\right| + \sin^2(\theta_{12})\left|\Delta M^2_{32}\right|. \tag{2}$$

On the other hand, by the 16th peak P_{13} and P_{23} are in phase, and so at energies as low as E_{16} the peak locations are instead roughly those of 2-flavor oscillation with a mass of $|\Delta M^2_{31}|$, which is greater (less) than ΔM^2_{eff} if the hierarchy is normal (inverted). If the hierarchy is normal then $|\Delta M^2_{31}|$ will be greater than ΔM^2_{eff} and so the energies E_n of the low energy peaks, corresponding to n well above 10, will be higher than would be obtained from a simple periodic extrapolation of the high energy ($n \leqslant 10$) peaks. For example, fixing M^2_{eff}, E_{16} would be about 2% higher in the case of the normal hierarchy.

Clearly such an experiment needs to be able to measure the energy with a precision much better than 2%. The energy of the neutrino cannot be measured directly, but the energy of the positron resulting from the inverse β decay is determined by counting photoelectrons in a photomultiplier. At low energies, not many of these photoelectrons are detected and so statistical fluctuations in the number of photoelectrons limit the energy resolution. As a result, the low energy peaks, which anyway are closer together, are smeared. Assuming about 1200 photoelectrons/MeV of prompt energy (the positron plus the electron with which it annihilates), which is about the most which can be hoped for with an organic liquid scintillator, the energy resolution will be about 3% times the square root of the prompt energy in MeV. With this resolution, our simulations indicate that for n greater than about 17, the identification of an individual peak is hopeless. The same simulations show that a determination of the hierarchy at a reactor experiment using these methods requires the observation of at least the $(|\Delta M^2_{31}|/\Delta M^2_{21} - 1)$st peak, which with the current best fit parameters corresponds to the 14th peak. Therefore only a modest reduction of the energy resolution, an increase in $|\Delta M^2_{31}|$ or a decrease in ΔM^2_{21} can destroy the ability to determine the hierarchy at such an experiment, as was reported in Ref. [12].

On the other hand, the high energy peaks which determine ΔM^2_{eff} can be reliably measured. As a result, such experiments can easily measure ΔM^2_{eff}, the main difficulty in the determination of the hierarchy comes from the low energy measurement of $|\Delta M^2_{31}|$. In particular, since all of the peaks $n \leqslant 10$ measure the same quantity ΔM^2_{eff}, little is gained by considering the peaks in the high energy tail of the reactor neutrino spectrum.

All experimental analyses that determine the hierarchy solely from reactor neutrinos rely upon the breakdown in periodicity described above. Two kinds of analysis have been studied extensively in the literature. First, one may perform a χ^2 fit to the observed spectra assuming both hierarchies, and conclude that the hierarchy is the one which minimizes χ^2. This method suffers from the fact that there are many nuisance parameters which need to be considered in the determinations of the spectra, and it would be impractical to extremize χ^2 with respect to all of them. As a result, a simpler method has been proposed in Ref. [13], in which one considers a Fourier transform of the observed spectrum and identifies several hierarchy-dependent quantities associated to the transformed spectrum which are reasonably independent of some of these nuisance parameters. More such properties were identified in Refs. [10,14] and applied to simulated data in Ref. [15].

Which method is better? Refs. [16,17] and [12] have shown that both methods yield reasonably consistent determinations of the mass hierarchy. As has been shown in Refs. [16,18,19] much can be gained by incorporating data from other experiments. It is currently only known how to do this with the χ^2 approach. On the other hand, a χ^2 approach requires a quantification of all of the effects which enter into the spectrum, such as broad modifications to the reactor flux coming from weak magnetism, the detector's nonlinear energy response and various reasonably smooth backgrounds. At this point even the size of the errors on some of these effects cannot be reliably estimated [20] and so spurious dependences will necessarily arise.

On the other hand the Fourier approach only considers a part of the information available, eliminating these spurious dependences but at the same time leading to an inherent inefficiency. A χ^2 fit to the image of a handwritten number or to the position-space cosmic microwave background (CMB) temperature would require so many nuisance parameters that it would be useless, which is why Fourier transforms are used to truncate the useless information out of the CMB. As a result, the Fourier and χ^2 analyses are complimentary to each other. Just as the Daya Bay experiment analyzed their data with 5 different methods to be sure that their result is analysis-independent, we expect that JUNO and RENO 50 will both analyze their data using both the χ^2 and Fourier approaches. Thus it is important to understand the drawbacks of each approach and how they can be resolved.

In Ref. [17], the authors observed that, using a Fourier transform based-analysis, the hierarchy-dependent quantities, contrary to their original motivation, are extraordinarily sensitive to the neutrino mass differences and also to the model of the reactor spectrum. We will now will explain the origin of this sensitivity.

As the dependences of the various quantities are virtually indistinguishable, for brevity we will consider only [14]

$$RL = \frac{R-L}{R+L}, \tag{3}$$

which is the fractional difference between two minima R and L of the Fourier cosine transform of the neutrino spectrum

$$F_c(k) = \int d\left(\frac{L}{E}\right)\frac{E^2}{L}\frac{\Phi(E)\sigma(E)}{4\pi L^2}P_{ee}\left(\frac{L}{E}\right)\cos\left(\frac{kL}{E}\right), \tag{4}$$

where E is the neutrino's energy and the tree level neutrino inverse β decay cross section is [21]

$$\sigma(E) = 0.0952 \times 10^{-42}\ \text{cm}^2 \frac{E_e\sqrt{E_e^2 - m_e^2}}{\text{MeV}^2},$$
$$E_e = E - m_n + m_p. \tag{5}$$

A $3\%/\sqrt{(E_e + m_e)/\text{MeV}}$ energy resolution is included by convoluting the observed energy spectrum with

$$\exp\left(-\frac{(E-E')^2}{0.0018(E_e+m_e)\ \text{MeV}}\right). \tag{6}$$

The masses of the electron, proton and neutron are m_e, m_p and m_n. We use the neutrino mass matrix parameters

$$\sin^2(2\theta_{13}) = 0.092, \qquad \sin^2(2\theta_{12}) = 0.861, \qquad \sin^2(2\theta_{32}) = 1,$$
$$\Delta M^2_{21} = 7.59 \times 10^{-5}\ \text{eV}^2, \qquad \left|\Delta M^2_{32}\right| = 2.43 \times 10^{-3}\ \text{eV}^2, \tag{7}$$

where $\sin^2(2\theta_{13})$ is that of Ref. [1], $\sin^2(2\theta_{12})$ and ΔM^2_{21} are taken from Ref. [22] and $|\Delta M^2_{32}|$ is that of Ref. [23]. These are not the

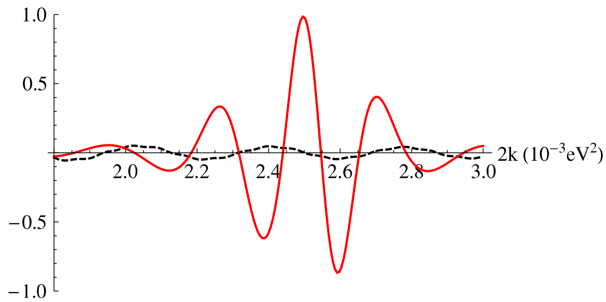

Fig. 1. The cosine transforms of the unoscillated reactor flux (black dashed curve) and the full $P_{13}+P_{23}$ oscillated flux (red solid curve) are shown. Note that the amplitude of the reactor flux oscillations is not much smaller than the difference between the depths of the two minima of $P_{13}+P_{23}$, which yields the hierarchy signal *RL*. For the value of $\Delta M^2_{\rm eff}$ drawn here, the reactor flux attains a maximum at the shallowest (left side) minimum of $P_{13}+P_{23}$, and so its contribution enhances *RL*, artificially inflating the confidence of the hierarchy determination. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

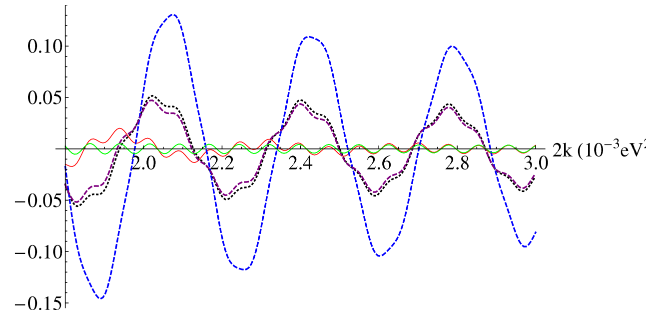

Fig. 2. The cosine transforms of the unoscillated flux is shown for numerically interpolated fluxes from the 1980s [24–26] (black dotted curve), for a quadratic fit to fluxes from the 1980s [26] and for quintic fits of the new fluxes of Ref. [20]. The latter two are shown with cutoffs of 8.5 MeV (dashed curves) and 12.8 MeV (solid curves). The blue dashed curve corresponds to the quadratic fit flux. The red and green solid curves, corresponding to 12.8 MeV cutoffs, are close to zero. Therefore the interference effect is present if the cutoff is at 8.5 MeV and but not if the fits are naively extrapolated to 12.8 MeV. This demonstrates that *RL* is sensitive to the neutrino spectrum above 8.5 MeV. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

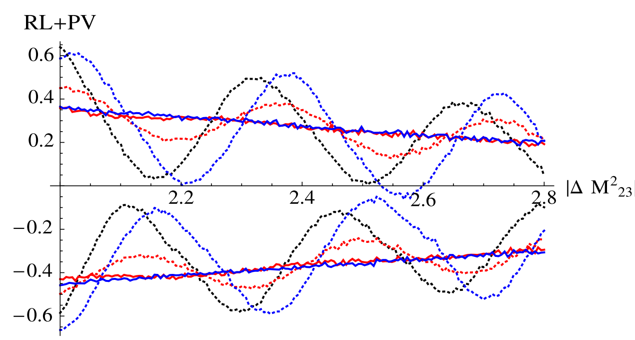

Fig. 3. Simulated average values of $RL+PV$ obtained from 100 000 neutrinos observed at a baseline of 58 km for various reactor flux models and mass differences $|\Delta M^2_{32}|$. The black dotted curve corresponds to the numerically interpolated fluxes from the 1980s [24–26], the dotted (solid) blue curve corresponds to a quadratic fit to fluxes from the 1980s [26] cut off at 8.5 MeV (12.8 MeV) and the dotted (solid) red curves to the quintic fits of the new fluxes of Ref. [20] cut off at 8.5 MeV (12.8 MeV). Notice that an unphysical extrapolation to 12.8 MeV eliminates the oscillations, therefore the oscillations result from the high energy part of the spectrum. $|\Delta M^2_{32}|$ is reported in units of 10^{-3} eV^2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

most recent values available, for example KamLAND has provided a more precise determination of ΔM^2_{21} and both MINOS and Daya Bay have provided more precise determinations of various combinations of $|\Delta M^2_{32}|$ and $|\Delta M^2_{31}|$. These rather old values are chosen for easy comparison with similar studies in the literature. As was found in Ref. [12] this small shift in the values of the parameters has little affect on our results.

As was demonstrated in Ref. [10], the minima whose difference defines *RL* lie just on either side of $k=|\Delta M^2_{31}|/2$. These minima arise from the Fourier transform of P_{13} which is independent of the hierarchy, but the contribution of P_{23} provides a perturbation which makes the right (left) minimum deeper for the normal (inverted) hierarchy. Thus the authors concluded that the hierarchy can be well determined by simply observing which of the two minima is deeper.

In Ref. [17] the authors described a problem with this approach. They observed that, depending upon the reactor flux model used, the transform of the unoscillated reactor flux $\Phi(E)\sigma(E)E^2/L^3$ itself may contribute near $k=|\Delta M^2_{31}|/2$, interfering with $P_{13}+P_{23}$ and so affecting *RL*. The unoscillated reactor flux contains no information about neutrino masses or the hierarchy, this contribution therefore contaminates the hierarchy-dependent observable *RL*. Thus *RL* does indeed depend upon the reactor flux model. But in Ref. [17] the authors also observed that this dependence is highly sensitive to the mass splitting, why is this?

While the cosine transform of the unoscillated flux $\Phi(E)\sigma(E)E^2/L^3$ is itself independent of the neutrino mass splittings, the locations of the peaks of $P_{13}+P_{23}$ are proportional to $\Delta M^2_{\rm eff}$. This means that the relative phase between the Fourier transform of the unoscillated spectrum and that of $P_{13}+P_{23}$ depends on the precise value of $\Delta M^2_{\rm eff}$. For example, for some values of $\Delta M^2_{\rm eff}$ the maximum of the Fourier transformed unoscillated reactor flux is coincident with the left (right) minimum of the transform of $P_{13}+P_{23}$, so this contribution increases (decreases) RL. As a result the oscillations in the Fourier transform of $\Phi(E)\sigma(E)$ lead to an $\Delta M^2_{\rm eff}$-dependence in the quantity *RL* just of the kind observed in Ref. [17] using old reactor flux models.

In fact, using the ^{235}U flux from Ref. [24], the ^{239}Pu and ^{241}Pu fluxes from [25] and the Gaussian approximated ^{238}U flux from Ref. [26] with the isotope ratios of Ref. [14] we find an oscillation in the unoscillated spectrum term in Eq. (4). Using this old model of the reactor flux, in Fig. 1 we compare the Fourier transform of the unoscillated term with that of the $P_{13}+P_{23}$ term, which is sensitive to the hierarchy. One can see that the unoscillated term is periodic with the same wavelength as was observed in Fig. 4 of Ref. [17], and thus the interference between these two terms oscillates as $\Delta M^2_{\rm eff}$ varies, shifting the $P_{13}+P_{23}$ peaks and so reproducing the effect reported in that note.

Ref. [17] concludes that this strong dependence of *RL* upon the reactor flux means that a precise knowledge of this flux is desirable to determine the neutrino mass hierarchy at a short baseline experiment. Our conclusions differ, we claim that this apparent dependence on the reactor model and the mass splittings is merely a dependence upon the high energy tail of the spectrum. This can clearly be seen to be the case in Fig. 2, in which we plot the cosine transforms of the unoscillated reactor flux for several different reactor models with various high energy cutoffs. The oscillations are large for *every* reactor flux model with a cutoff of 8.5 MeV, but for none with a cutoff at 12.8 MeV. Thus, the large oscillations of the Fourier transform of the unoscillated spectrum are caused almost entirely by the very high energy tail of the spectrum, above 8.5 MeV, where there are few events and essentially no information regarding the hierarchy.

To demonstrate that these fluctuations in the cosine transform of the unoscillated reactor spectrum are indeed the culprit behind the spurious mass splitting dependence in Ref. [17], in our Fig. 3

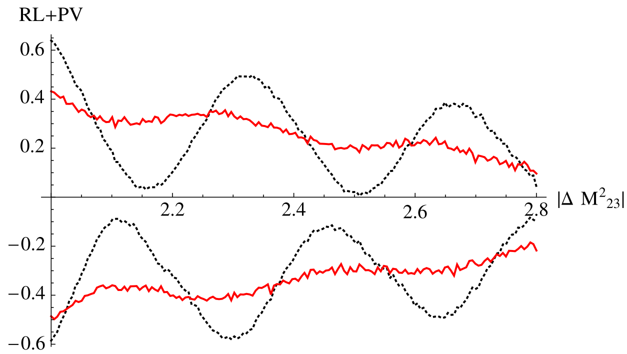

Fig. 4. Simulated average values of $RL + PV$ obtained from 100 000 neutrinos observed at a baseline of 58 km assuming the numerically interpolated reactor spectra from the 1980s [24–26]. The black dotted curve uses an unweighted Fourier transform and the red solid curve uses the weight $\exp(-0.04E^2/\mathrm{MeV}^2)$. $|\Delta M^2_{32}|$ is reported in units of 10^{-3} eV^2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

we reproduce their Fig. 4. This figure shows RL determined from simulated data with various mass splittings, reactor models and cutoffs. We find that the spurious dependence on the mass splitting in fact exists for *all* flux models, but for each model it goes away with a naive extrapolation of the corresponding flux fitting function up to 12.8 MeV. Indeed, we see that even for the old quadratic fits the oscillation can be removed for such a naive extrapolation.

Why does a cutoff change the value of RL? The reason is that RL depends on the shape of the Fourier transform at $k \sim |\Delta M^2_{31}|$. A general property of Fourier transforms is that any feature of the original spectrum with a width δ affects the Fourier transform at modes $k \sim 1/j\delta$ where j is an integer labeling the higher harmonics. For example, large delocalized changes in the spectrum such as a change in flux models correspond to large values of δ and so only affect the Fourier transform at very small values of k, and so leave RL invariant as was shown in Ref. [12]. On the other hand, a sharp cutoff corresponds to small value of δ and so the $j = 1$ mode $k \sim 1/\delta$ is quite large. However, the mode $j \sim 1/\delta|\Delta M^2_{31}|$ will affect the Fourier transform at $k \sim |\Delta M^2_{31}|$ and so affects RL. For δ sufficiently small, there will exist an integer j such that $j \sim 1/\delta|\Delta M^2_{31}|$. It will be important below that if one considers a soft cutoff so that $\delta > |\Delta M^2_{31}|$ then $1/\delta|\Delta M^2_{31}| < 1$ and so it is not equal to any integer j, therefore a soft cutoff does not affect the Fourier transform in the crucial region $k \sim |\Delta M^2_{31}|$ which determines RL.

Similarly, nuisance parameters and known effects with soft energy dependences do not affect the Fourier transform analysis, which is the original motivation for the Fourier transform technique. Examples of such parameters and effects include the weak magnetism effects in the reactor neutrino spectra and in its inverse β decay capture cross section, as well as one loop corrections in the latter. These corrections are described in Ref. [27], where it can be seen that the energy dependence is soft enough that they will have no effect at wavenumbers $k \sim |\Delta M^2_{31}|$ and so on the hierarchy determination using the Fourier method. However some nuisance parameters may be associated with small length scales, such as narrow geoneutrino spectra. These in principle can affect the Fourier analysis, although in the case of geoneutrinos it has been shown in Ref. [12] that, due to the low geoneutrino flux, this effect is negligible.

This does not imply that the problem observed in Ref. [17] can simply be eliminated by not cutting off the spectrum at high energies. The problem with this approach is that the flux fitting functions extrapolated up to 12.8 MeV in many previous studies do not provide good approximations to the reactor flux at these high energies. On the contrary, as was noted in Ref. [26] the extrapolation of the quadratic flux fit to 12.8 MeV yields 2 to 3 times more flux than is observed above about 8.5 MeV, and so therefore is unphysical. Thus although the real reactor neutrino flux does not exhibit a hard cut off at 8.5 MeV, a hard cut off at 8.5 MeV may nonetheless provide as good of an approximation to the true spectrum as no cut off at all. The quintic fit, as it is based on a fit to data at energies below 8.5 MeV, has a similar problem.

From this analysis we learn two lessons. First of all, as the real reactor neutrino flux above 8.5 MeV is not well approximated by the simple fitting functions which work at lower energies, one can expect that the Fourier transform of the true spectrum will exhibit the oscillations described above and so contaminate RL and $RL+PV$ in the manner observed in Ref. [17]. Second, this effect is missed in a simulation which naively extrapolates these fitting functions well beyond 8.5 MeV, perhaps explaining why it had not been observed in earlier studies.

As RL depends strongly on the spectrum between 8.5 *and* 12.8 MeV, *which in turn is independent of the hierarchy, this high energy tail provides a nuisance parameter for the determination of the hierarchy using RL.* The solution suggested in Ref. [17] is to determine the spectrum precisely, however so few neutrinos are observed in this range that such a determination would be difficult, indeed the spectrum is not understood at the required precision even at the energies with high fluxes [28]. Even if such a measurement were possible, then RL would still depend upon $\Delta M^2_{\mathrm{eff}}$ with a higher sensitivity than the mass determination at MINOS, making a determination of the hierarchy at a medium baseline more challenging.

Our solution is to replace RL and PV with quantities that are insensitive to the high energy neutrino spectrum, by providing an energy-dependent weight $w(E)$ on the neutrino spectrum in the Fourier transform. As we saw in Fig. 2, a simple cutoff in the Fourier transform will amplify the spurious dependence. The weight needs to cut off the high energies gradually, with derivative scales much longer than $|\Delta M^2_{31}|$, so as to not itself introduce spurious peaks in the critical part of the Fourier transforms. One such choice of weight which we have found works quite well is a Gaussian

$$F_c(k) = \int d\left(\frac{L}{E}\right) e^{-\frac{0.04E^2}{\mathrm{MeV}^2}} \frac{E^2}{L} \frac{\Phi(E)\sigma(E)}{4\pi L^2} P_{ee}\left(\frac{L}{E}\right) \cos\left(\frac{kL}{E}\right). \quad (8)$$

The same weight serves well in both the sine transform and also the nonlinear transforms of Ref. [10] which determine the hierarchy more reliably than $RL + PV$ at baselines below about 55 km [29]. In Fig. 4 we use simulated data to compare the value of $RL + PV$ obtained from the ordinary Fourier transform with that obtained using the weighted Fourier transform. The oscillations almost disappear in the weighted case.

As can be seen by comparing the unweighted and weighted cosine transforms in Figs. 1 and 5, not only does the weighting procedure preserve RL, but given enough detected neutrinos it actually increases the difference in the peak sizes between the normal and inverted hierarchies. Thus this solution to the dependence upon the high energy neutrino tail not only removes the spurious dependence, for any high energy reactor spectrum, but it can even increase the chance of success of the determination of the hierarchy. This benefit is greatest when a large number of neutrinos is detected, since the weighting effectively reduces the statistics at high energies. In Refs. [12,29] we use the weighted Fourier transform to analyze simulated data.

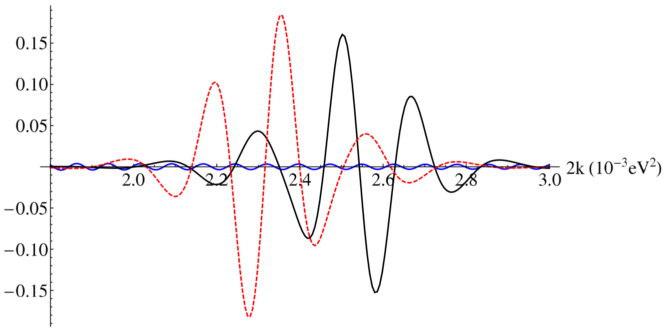

Fig. 5. Here we see the theoretical weighted Fourier transform of the spectrum without oscillations (blue solid curve) and with oscillations in the case of the normal (black solid curve) and inverted (red dashed curve) hierarchies. One can see that the solid, blue unoscillated curve is very close to zero. We have checked that this curve is essentially independent of the cutoff and so the reactor spectrum no longer affects *RL*. Comparing with Fig. 1 one can see that the difference *RL* between the depths of the minima is even greater in this weighted case, allowing for a better determination of the hierarchy than was possible with an unweighted Fourier transform. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Acknowledgements

J.E. is supported by the Chinese Academy of Sciences Fellowship for Young International Scientists grant number 2010Y2JA01. E.C. and X.Z. are supported in part by the NSF of China.

References

[1] F.P. An, et al., DAYA-BAY Collaboration, Observation of electron-antineutrino disappearance at Daya Bay, Phys. Rev. Lett. 108 (2012) 171803, arXiv:1203.1669 [hep-ex].

[2] D. Dwyer, Daya Bay Results, presented at Neutrino 2012 in Kyoto. Available at http://neu2012.kek.jp/neu2012/programme.html.

[3] J.K. Ahn, et al., RENO Collaboration, Observation of reactor electron antineutrino disappearance in the RENO experiment, Phys. Rev. Lett. 108 (2012) 191802, arXiv:1204.0626 [hep-ex].

[4] M. Wilking, The latest results from T2K on the neutrino oscillation and interactions, presented at EPS 2013 in Stockholm. Available at http://indico.cern.ch/conferenceDisplay.py?confId=218030.

[5] S.T. Petcov, M. Piai, The LMA MSW solution of the solar neutrino problem, inverted neutrino mass hierarchy and reactor neutrino experiments, Phys. Lett. B 533 (2002) 94, arXiv:hep-ph/0112074;
S. Choubey, S.T. Petcov, M. Piai, Precision neutrino oscillation physics with an intermediate baseline reactor neutrino experiment, Phys. Rev. D 68 (2003) 113006, arXiv:hep-ph/0306017.

[6] J. Cao, Observation of $\bar{\nu}_e$ Disappearance at Daya Bay, presented at νTurn under Gran Sasso. Available at http://agenda.infn.it/contributionListDisplay.py?confId=4722.

[7] Observation of reactor neutrino disappearance at RENO, presented at νTURN 2012 under Gran Sasso. Available at http://agenda.infn.it/contributionListDisplay.py?confId=4722.

[8] Y. Wang, Daya Bay II: The next generation reactor neutrino experiment, presented at NuFact in Williamsburg, Virginia. Available at https://www.jlab.org/indico/conferenceTimeTable.py?confId=0#20120725.detailed.

[9] H. Minakata, H. Nunokawa, S.J. Parke, R. Zukanovich Funchal, Determination of the neutrino mass hierarchy via the phase of the disappearance oscillation probability with a monochromatic anti-electron-neutrino source, Phys. Rev. D 76 (2007) 053004, arXiv:hep-ph/0701151;
H. Minakata, H. Nunokawa, S.J. Parke, R. Zukanovich Funchal, Phys. Rev. D 76 (2007) 079901 (Erratum).

[10] E. Ciuffoli, J. Evslin, X. Zhang, The neutrino mass hierarchy at reactor experiments now that theta13 is large, J. High Energy Phys. 1303 (2013) 016, arXiv:1208.1991 [hep-ex].

[11] H. Nunokawa, S.J. Parke, R. Zukanovich Funchal, Another possible way to determine the neutrino mass hierarchy, Phys. Rev. D 72 (2005) 013009, arXiv:hep-ph/0503283.

[12] E. Ciuffoli, J. Evslin, X. Zhang, Optimizing medium baseline reactor neutrino experiments, Phys. Rev. D 88 (2013) 033017, arXiv:1302.0624 [hep-ph].

[13] G. Varner, J. Learned, S.T. Dye, S. Pakvasa, R.C. Svoboda, Determination of neutrino mass hierarchy and θ_{13} with a remote detector of reactor antineutrinos, Phys. Rev. D 78 (2008) 071302, arXiv:hep-ex/0612022;
M. Batygov, S. Dye, J. Learned, S. Matsuno, S. Pakvasa, G. Varner, Prospects of neutrino oscillation measurements in the detection of reactor antineutrinos with a medium-baseline experiment, arXiv:0810.2580 [hep-ph].

[14] L. Zhan, Y. Wang, J. Cao, L. Wen, Determination of the neutrino mass hierarchy at an intermediate baseline, Phys. Rev. D 78 (2008) 111103, arXiv:0807.3203 [hep-ex].

[15] L. Zhan, Y. Wang, J. Cao, L. Wen, Experimental requirements to determine the neutrino mass hierarchy using reactor neutrinos, Phys. Rev. D 79 (2009) 073007, arXiv:0901.2976 [hep-ex].

[16] P. Ghoshal, S.T. Petcov, Neutrino mass hierarchy determination using reactor antineutrinos, J. High Energy Phys. 1103 (2011) 058, arXiv:1011.1646 [hep-ph].

[17] X. Qian, D.A. Dwyer, R.D. McKeown, P. Vogel, W. Wang, C. Zhang, Mass hierarchy resolution in reactor anti-neutrino experiments: parameter degeneracies and detector energy response, Phys. Rev. D, Part. Fields 87 (2013) 033005, arXiv:1208.1551 [physics.ins-det].

[18] Y.-F. Li, J. Cao, Y. Wang, L. Zhan, Unambiguous determination of the neutrino mass hierarchy using reactor neutrinos, Phys. Rev. D 88 (2013) 013008, arXiv:1303.6733 [hep-ex].

[19] E. Ciuffoli, J. Evslin, X. Zhang, Confidence in a neutrino mass hierarchy determination, arXiv:1305.5150 [hep-ph].

[20] P. Huber, On the determination of anti-neutrino spectra from nuclear reactors, Phys. Rev. C 84 (2011) 024617, arXiv:1106.0687 [hep-ph];
P. Huber, Phys. Rev. C 85 (2012) 029901 (Erratum).

[21] P. Vogel, J.F. Beacom, Angular distribution of neutron inverse beta decay, antineutrino $e^- + p \to e^+ + n$, Phys. Rev. D 60 (1999) 053003, arXiv:hep-ph/9903554.

[22] B. Aharmim, et al., SNO Collaboration, Low energy threshold analysis of the phase I and phase II data sets of the Sudbury neutrino observatory, Phys. Rev. C 81 (2010) 055504, arXiv:0910.2984 [nucl-ex].

[23] P. Adamson, et al., MINOS Collaboration, Measurement of neutrino oscillations with the MINOS detectors in the NuMI beam, Phys. Rev. Lett. 101 (2008) 131802, arXiv:0806.2237 [hep-ex].

[24] K. Schreckenbach, G. Colvin, W. Gelletly, F. Von Feilitzsch, Determination of the antineutrino spectrum from U-235 thermal neutron fission products up to 9.5 MeV, Phys. Lett. B 160 (1985) 325.

[25] A.A. Hahn, K. Schreckenbach, G. Colvin, B. Krusche, W. Gelletly, F. Von Feilitzsch, Anti neutrino spectra from Pu-241 and Pu-239 thermal neutron fission products, Phys. Lett. B 218 (1989) 365.

[26] P. Vogel, J. Engel, Neutrino electromagnetic form-factors, Phys. Rev. D 39 (1989) 3378.

[27] P. Vogel, Analysis of the anti-neutrino capture on protons, Phys. Rev. D 29 (1984) 1918.

[28] G. Mention, M. Fechner, T. Lasserre, T.A. Mueller, D. Lhuillier, M. Cribier, A. Letourneau, The reactor antineutrino anomaly, Phys. Rev. D 83 (2011) 073006, arXiv:1101.2755 [hep-ex];
E. Ciuffoli, J. Evslin, H. Li, The reactor anomaly after Daya Bay and RENO, arXiv:1205.5499 [hep-ph].

[29] E. Ciuffoli, J. Evslin, X. Zhang, Mass hierarchy determination using neutrinos from multiple reactors, J. High Energy Phys. 1212 (2012) 004, arXiv:1209.2227 [hep-ph].